NUMERICAL PROBLEMS

10.1 The time period of a simple pendulum is 2s. What will be its length on Earth? What will be its length on the moon if $g_m = g_e /6$? Where $g_e = 10ms^{-2}$.

Answer

$T = 2Sec$

$l_e = ?$

$l_m = ?$

$g_e = 10ms^{-2}$

$g_m = \frac{g_e}{6} = 1.6ms^{-2}$

a) For earth,

$T = 2\pi\sqrt{\frac{l_e}{g_e}}$

or $T^2 = 4\pi^2\left(\frac{l_e}{g_e}\right)$

$l_e = \frac{T^2 \times 10}{4\pi^2}$

or $\boxed{l_e = 1.02\ m}$

a) For moon,

$T = 2\pi\sqrt{\frac{l_m}{g_m}}$

$T^2 = 4\pi^2\left(\frac{l_m}{g_m}\right)$

$l_e = \frac{T^2 \times g_m}{4\pi^2}$

As the time period is same;

So, $l_m = \frac{2^2 \times 1.6}{4\pi^2}$

$\boxed{l_m = 0.17\ m}$

10.2 A pendulum of length 0.99 m is taken to the moon by an astronaut. The period of the pendulum is 4.9s. What is the value of g on the surface of the moon?

Answer

$l_m = 0.99\ m$

$T_m = 4.9\ Sec$

$g_m = \ ?$

$\Rightarrow \quad T_m = 2\pi\sqrt{\frac{l_m}{g_m}}$

$\Rightarrow \quad T_m^2 = 4\pi^2\sqrt{\frac{l_m}{g_m}}$

$g_m = \frac{4\pi^2 \times 0.99}{(4.9)^2}$

$\boxed{g_m = 1.63\ ms^{-2}}$

10.3 Find the time periods of a simple pendulum of 1-meter length, placed on Earth and on moot). The value of g on the surface of moon is 1/6th of its value on Earth. Where g_e is 10ms^{-1}.

$T_e = Tm = ?$

$\Rightarrow \quad l = 1m$

$g_e = 10ms^{-2}$

$g_m = 1.67ms^{-2}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{l_e}{g_e}}$ (As the length is same on earth and moon)

$T_e = 2\pi\sqrt{\frac{1}{10}}$

$T_e = 2sec$

and, $T_m = 2\pi\sqrt{\frac{l_m}{g_m}}$

$T_m = 2\pi\sqrt{\frac{1}{1.87}}$

$\boxed{T_m = 4.9\ Sec}$

10.4 A simple pendulum completes one vibration in two seconds. Calculate its length when g 10.0ms^{-2}

$l = 1m$

$T = ?$

$g = 10ms^{-2}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{g}}$

$T^2 = 4\pi^2\left(\frac{l}{g}\right)$

$l = \frac{T^2 \times g}{4\pi^2}$

$l = \frac{2^2 \times 10}{4\pi^2}$

$\boxed{l = 1.02\ m}$

10.5 If 100 waves pass through a point of a medium in 20 seconds, what is the frequency and the time period of the wave? If its wavelength is 6cm, calculate the wave speed.

Answer

$f = ?$ $n = 100,\ t = 20 Sec$

$T = ?$

$\lambda = 6cm = 0.06m$

We know that, $f = \frac{n}{t}$

$= \frac{100}{20}$

$\boxed{f = 5 \text{ Hz}}$

$T = \frac{1}{f}$

$T = \frac{1}{5}$

$\boxed{T = 0.2 \text{ Sec}}$

$v = ?$

$\Rightarrow$ $v = f\lambda$

$= 5 \times 0.06$

$\boxed{v = 0.3 \text{ ms}^{-1}}$

10.6 A wooden bar vibrating into the water surface in a ripple tank has a frequency of 12 Hz. The resulting wave has a wavelength of 3 cm. What is the speed of the wave?

Answer

$f = 12$ Hz

$\lambda = 3$ cm

$= 0.03$ m

$v = ?$

We know that $v = f\,\lambda$

$= 12 \times 0.03$

$\boxed{v = 0.36\ ms^{-1}}$

10.7 A transverse wave produced on a spring has a frequency of 190 Hz and travels along the length of the spring of 90 m, in 0.5 s.

a) What is the period of the wave?

b) What is the speed of the wave?

c) What is the wavelength of the wave?

$f = 190\ Hz$

$T = ?$

$v = ?\ \lambda = ?$

We know that, $T = \frac{1}{f}$

$= \frac{1}{190}$

$T = 0.005\ Sec$

or $\boxed{T = 0.01\ Sec}$

$v = \frac{d}{t}, \frac{90}{0.5}$

$\boxed{v = 180\ ms^{-1}}$

$\Rightarrow \quad v = f\,\lambda$

$\lambda = \frac{v}{f}$

$\lambda = \frac{180}{190}$

$\boxed{\lambda = 0.95m}$

10.8 Water waves in a shallow dish are 6.0 cm long. At one point, the water moves up and down at a rate of 4.8 oscillations per second.

a) What is the speed of the water waves?

b) What is the period of the water waves?

Answer

$\lambda = 6$ cm

$= 0.06$ m

$f = 4.8$ Hz

$V = ?$

$T = ?$

As we know that, $V = f\lambda$

$v = 4.8 \times 0.06$

$\boxed{v = 0.29\ ms^{-1}}$

$T = \frac{1}{f}$

$T = \frac{1}{4.8}$

$\boxed{T = 0.21\ Sec}$

10.9 At one end of a ripple tank 80 cm across, a 5 Hz vibrator produces waves whose wavelength is 40 mm. Find the time the waves need to cross the tank.

Answer

$f = 5$ Hz

$\lambda = 40$ mm

$= 0.04$ m

$t = ?$

We know that $v = f\lambda$

$= 5 \times 0.04$

$\boxed{v = 0.2\ ms^{-1}}$

$\Rightarrow \quad v = \frac{d}{t}$

$= \frac{0.8}{0.2}$

$\boxed{v = 4\ Sec}$

10.10 what is the wavelength of the radio waves transmitted by an FM station at 90 MH_{z} where 1M =10^{6} and speed of radio wave is 3x 10^{8} ms^{-1}.

Answer

$\lambda = ?$

$f = 90$ MHz

$f = 90 \times 10^{6}$ HZ

$V = 3 \times 10^{8}\ ms^{-1}$

$\Rightarrow \quad v = f\lambda$

$\lambda = \frac{v}{f}$

$\lambda = \frac{3 \times 10^{8}}{90 \times 10^{6}}$

$\boxed{\lambda = 3.33m}$

Numerical Problems

11.1 A normal conversation involves sound intensities of about 3.0×10^{-6} Wm^{-2}. What is the decibel level for this intensity? What is the intensity of the sound for 100dB?

Answer

$I = 3.0 \times 10^{-6}$ Wm^{-2}

Sound level =?

Then (Sound level) = 100dB.

$I' =?$

For the 1st part, we have,

Sound level = $10 \log \left(\frac{I}{I_0}\right)$ dB

$= 10 \log \left(\frac{3 \times 10^{-6}}{10^{-12}}\right)$

because, the faintest intensity of sound is; $I_0 = 10^{-12}$ Wm^{-2}

⇒ Sound level = $10 \log (3 \times 10^{6})$ dB

or $\boxed{\text{Sound level} = 64.8 \text{ dB}}$

For the second part, we have,

(Sound level) = 10 log

$= 10 \log \left(\frac{I'}{I_0}\right)$ dB.

$100 = 10 \log \left(\frac{I'}{10^{-12}}\right)$ dB

or $10 = \log \left(\frac{I'}{10^{-12}}\right)$

As given in mathematics; $\log\left(\frac{a}{b}\right) = \log a - \log b$

So, $10 = \log I' - \log(10^{-12})$

Also, according to logarithm rules, $\log(10)^a = a \log(10)$

So, $10 = \log I' - (-12)\log(10)$

As, $\log 10 = 1$

So, $10 = \log I' + 12$

$\log I' = -2$

Taking antilog;

$I' = \text{Antilog}(-2)$

$\boxed{I' = 0.01\ Wm^{-2}}$

11.2 If at Anarkali bazaar Lahore, the sound level is 80 dB, what will be the intensity level of sound there?

Answer

Sound level = 80 dB

Intensity level = I =?

Intensity of faintest sound, I_0 We have, = $10^{-12}\ Wm^{-2}$

Sound level = $10 \log\left(\frac{I}{I_0}\right)$ dB

or $80 = 10 \log\left(\frac{I}{10^{-12}}\right)$ dB

$8 = \log I - \log(10^{-12})$

$8 = \log I - (-12)\log(10)$

$8 = \log I + 12$

$\log I = -4$

Taking antilog we have,

$$I = \text{Antilog}(-4)$$

$$\boxed{I = 10^{-4}\,Wm^{-2}}$$

11.3 At a particular temperature, the speed of sound in air is 330 ms^-1. If the wavelength of a note is 5 cm, calculate the frequency of the sound wave. Does its frequency lie in the audible range of the human ear?

Answer

$v = 330\ ms^{-1}$

$\lambda = 5\ cm$

$= 0.05\ m$

$f = ?$

$v = f\lambda$

or $f = \frac{v}{\lambda}$

Putting the values

$f = \frac{330}{0.05}$

$$\boxed{f = 6.6 \times 10^{3}\ Hz}$$

11.4 A doctor counts 72 heartbeats in 1 min. Calculate the frequency and period of the heartbeats

Answer

Number of heart beats $n = 72$

Time, $t = 1$ min

$= 60$ Seconds

Time period, $T = ?$

frequency, $f = ?$

We have that, the time period is equal to number of vibrations per total time, and frequency is its inverse

So $f = \frac{n}{t}$

Putting the values.

$$f = \frac{72}{60}$$

$$\boxed{f = 1.2\ Hz}$$

$$f = \frac{1}{T}, \text{ or } T = \frac{1}{f}$$

(As frequency and time period are reciprocal to each other.)

$$\boxed{T = 0.83 \text{ Seconds}}$$

11.5 A marine survey ship sends a sound wave straight to the sea bed. It receives an echo 1.5 s later. The speed of sound in sea water is 1500 ms^{-1}. Find the depth of the sea at this position.

Answer

$$\left\{\begin{array}{c}\text{Time taken by sec and wave from} \\ \text{Ship to the sea bed and then back,} \\ \text{to ship}\end{array}\right. t = 1.5\text{sec.}$$

Time for one side (from ship to sea bed) $t' = \frac{1.5}{2}$

$= 0.75$ Sec

Speed of sound, $V = 1500\ ms^{-1}$

Depth of Sea, $d = ?$

We know that,

$$V = \frac{d}{t'}$$

or $d = v \times t'$

Putting the values, $d = 1500 \times 0.75$

$$\boxed{d = 1125\ m}$$

11.6 A student clapped his hands near a cliff and heard the echo after 5 s. What is the distance of the cliff from the student if the speed "of the sound, v is taken as 346 ms^{-1}?

Answer

$$\left\{\begin{array}{c}\text{Time taken Time taken by sound from} \\ \text{the student to cliff and then,} \\ \text{back to student}\end{array}\right. t = 5\text{sec.}$$

Time for one side (from cliff to student $t' = \frac{5}{2}$

$t' = 2.5$ Sec

Velocity, $V = 346\ ms^{-1}$

Distance from cliff to student, $d = ?$

We know that,

$$V = \frac{d}{t'}$$

or $d = v \times t'$

Putting the values, $d = 346 \times 2.5$

$\boxed{d = 865\ m}$

11.7 A ship sends out ultrasound that returns from the seabed and is detected after 3.42s. If the speed of ultrasound through sea water is 1531 ms^{-1}, what is the distance of the seabed from the ship?

Answer

$$\left\{\begin{array}{c}\text{Time taken by ultrasound from} \\ \text{Ship to the sea bed and then back, } t = 3.42\text{sec} \\ \text{to ship}\end{array}\right.$$

Time for one side (from ship to sea bed) $t' = \frac{3.42}{2}$

$= 1.71$ Sec

Speed of ultrasound, $V = 1531\ ms$

Distance of sea bed from the ship, d = ?

We know that,

$$V = \frac{d}{t'}$$

or $$d = v \times t'$$

Putting the values, $d = 1531 \times 1.71$

$$\boxed{d = 2618 \text{ m}}$$

11.8 The highest frequency sound humans can hear is about 20,000 Hz. What is the wavelength of sound in air at this frequency at a temperature of 20°C? What is the wavelength of the lowest sounds we can hear of about 20Hz? Assume the speed of sound in air at 20°C is 343 ms-1.

Answer

Highest sound frequency, f = 20,000 Hz

Wave length sound λ =?

Lowest sound frequency, f'= 20 Hz

Wave length of sound, λ' =?

For the first part

$$v = f\lambda$$

⇒ while V in both the cases will be same'

i.e $v = 343$ ms

So $\lambda = \frac{v}{f}$

= 34320000

$$\boxed{\lambda = 1.7 \times 10^{-2}\ m}$$

For the second part:

$v = f'\lambda'$

or $\lambda' = \frac{v}{f'}$

$= \frac{343}{20}$

$$\boxed{\lambda' = 17.2\ m}$$

11.9 A sound wave has a frequency of 2 kHz and wavelength 35 cm. How long will it take to travel 1.5 km?

Answer

frequency of sound waves, f = 2 kHz

or = 20000 Hz

Wave length, λ = 35 cm

= 0.35 m

Time taken t = ?

Distance covered, d = 1.5 km

= 1500 m

We know that $= v = \frac{d}{t}$

$t = \frac{d}{v}$ (I)

but velocity is wave is not known.

So $v = f\lambda$

$= 20000 \times 0.35$

$v = 700\ ms$

putting the values of velocity and distance in equation (1), we get

$t = \frac{d}{v}$

$= \frac{1500}{700} \Rightarrow \boxed{t = 2.1 \text{ seconds}}$

NUMERICAL PROBLEMS

12.1 An object 10.0 cm in front of a convex mirror forms in image 5.0 cm behind the mirror. What is the focal length of the mirror?

Answer

Distance of object, P = 10m

Distance of image = 5 cm

focal length f =?

As we know that,

$$\frac{1}{f} = \frac{1}{p} + \left(\frac{1}{q}\right)$$

Negative sign indicates that the image formed by convex mirror is virtual,

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{10} + \left(\frac{1}{5}\right)$$

$$= \frac{1-2}{10}$$

$$\frac{1}{f} = \frac{1}{10}$$

$$\boxed{f = -10 \text{ cm}}$$

12.2 An object 30.0 cm tall is located 10.5 cm from a concave mirror with focal length 16.0 cm. (a) where is the image located? (b) How high is it?

Answer

Distance of object, P - 10.5 cm

Focal length, f =16 cm

Height of object, $h_o = 30$ cm

Height of image, $h_i = ?$

Distance of image, $q = ?$

we know the formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \quad \text{or} \quad \frac{1}{q} = \frac{1}{f} \quad \frac{1}{p}$$

Putting the values, we get

$$\frac{1}{q} = \frac{1}{16} \quad \frac{1}{10.5}$$

or $$= \frac{1}{16} \quad \frac{10}{105}$$

$$= \frac{1}{16} \quad \frac{2}{21}$$

$$= \frac{21-32}{336}$$

$$\frac{1}{q} = \quad \frac{11}{336}$$

or $$q = \quad \frac{336}{11}$$

or $$\boxed{q = 30.54 \text{ cm}}$$

As only the magnitude is required, so

$$q = 30.54 \text{ cm}$$

Also, we know that,

$$\frac{\text{height of image}}{\text{height of object}} = \frac{\text{Distance of image}}{\text{Distance of object}}$$

$\Rightarrow$ $$\frac{h_i}{h_o} = \frac{p}{q}$$

or $$h = \frac{p}{q} \times h_o$$

$$= \frac{30.54}{10.5} \times 30$$

$$h = 87.26 \text{ cm}$$

⇒ Height of image, h_i = 87.26 cm

12.3 An object and its image in a concave mirror are of the same height, yet inverted, when the object is 20.0 cm from the mirror. What is the focal length of the mirror?

Answer

As it is given that

Height of image = height of object

=> $h_i = h_o$

Distance of object, p = 20 cm

Focal length, f =?

Let us find the distance of image 'q' first.

We know that,

$$\frac{h_i}{h_o} = \frac{q}{p}$$

or $q = \frac{h_i}{h_o} \times p$

As $h_i = h_o$

So $q = p$

or $\boxed{q = 20cm}$ (p =20cm)

Applying spherical mirror formula

i.e $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

⇒ $\frac{1}{f} = \frac{1}{20} + \frac{1}{20}$

$= \frac{1+1}{20}$

$\frac{1}{f} = \frac{2}{20}$

$f = 10cm$

12.4 Find the focal length of a mirror that forms an image 5.66 cm behind a mirror of an object placed at 34.4 cm in front of the mirror.

Answer

Focal length f =?

Distance of image, q = 5.66 cm

Distance of object, p = 34.4 cm

Applying spherical mirror formula:

i.e $\quad \frac{1}{f} = \frac{1}{p} + \frac{}{q}$

$= \frac{1}{34.4} \quad \frac{1}{5.66}$ (-ve sign is due to virtual image

$= \frac{1}{344} \quad \frac{1}{566}$

$= \frac{5}{172} \quad \frac{50}{283}$

$= \frac{1415 - 8600}{48676}$

$\frac{1}{f} = \frac{7185}{48676}$

$f = \frac{1876}{7185}$

$\boxed{f = 677cm}$

12.5 An image of a statue appears to be 11.5 cm behind a convex mirror with focal length 13.5 cm. Find the distance from the statue to the mirror

Answer

Distance of image, q = 11.5 cm

Focal length f =13.5 cm

Distance of object, p =?

Applying spherical mirror formula:

i.e $$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$$

$$= \frac{1}{13.5} - \frac{1}{-11.5}$$ (-ve sign is due to virtual image

$$= \frac{10}{135} + \frac{10}{115}$$

$$= \frac{2}{27} + \frac{2}{23}$$

$$= \frac{46+54}{621}$$

$$\frac{1}{p} = \frac{100}{621}$$

p = 621/100

$$\boxed{p = 6.2cm}$$

12.6 An image is produced by a concave mirror of focal length 8.70 cm. The object is 13.2 cm tall and at a distance 19.3 cm from the mirror, (a) Find the location and height of the image, (b) Find the height of the image produced by the mirror if the object is twice as far from the mirror.

Answer

Focal length, f = 8.7 cm

Distance of image, p = 19.3cm

Distance of object, q =?

Height of image, h_i =?

Height of object h_o = 13.2 cm

New distance of object, p = 38.6 cm (double of 19.3 cm)

New height of image h_1' =?

Applying spherical mirror formula,

i.e $$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$= \frac{1}{8.70} - \frac{1}{19.3}$$

$$= \frac{10}{870} - \frac{10}{193}$$

$$= \frac{10}{87} - \frac{10}{193}$$

$$= \frac{1930-870}{16791}$$

$$\frac{1}{q} = \frac{1060}{16791}$$

$$q = \frac{16791}{1060}$$

$$\boxed{q = 15.84 \text{ cm}}$$

Also we know that

$$\frac{h_1}{h_0} = \frac{q}{p}$$

or $$h_1 = \frac{q}{p} \times h_0$$

Putting the values, $h_1 = \frac{15.84}{19.3} \times 13.2$

$$\boxed{h_1 = 10.83 \text{ cm}}$$

When distance of object is made double then,

$$\frac{h'_1}{h_0} = \frac{q}{p'}$$

or $$h'_1 = \frac{q}{p'} \times h_0$$

$= \frac{15.84}{38.6} \times 13.2$

$\boxed{h_1 = 5.42cm}$

q will remain the same and it is formed at the same distance

12.7 Nabeela uses a concave mirror when applying makeup. The mirror has a radius of curvature of 38.0 cm. (a) What is the focal length of the mirror? (b) Nabeela is located 50 cm from the mirror, where will her image appear? (c) Will the image be upright or inverted?

Answer

Radius of curvature, R = 38 cm

Focal length, f =?

Distance of object, p = 50 cm

Distance of image, q =?

As we know that, f = R/2

So $f = \frac{38}{2}$

$f = 19$ cm

Applying spherical mirror formula

i.e $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$

Putting the values, we get

$\frac{1}{q} = \frac{1}{19} - \frac{1}{50}$

$= \frac{50-19}{950}$

$$\frac{1}{q} = \frac{31}{950}$$

$$q = \frac{950}{31}$$

or $\boxed{q = 30.64\ cm}$

(Image will be erect as we are viewing concave mirror)

12.8 An object 4 cm high is placed at a distance of 12 cm from a convex lens of focal length 8 cm. Calculate the position and size of the image. Also state the nature of the image.

Answer

Height of object, h_o = 4cm

Height of image, h_i =?

Distance of object, p = 12 cm

Focal length, f = 8 cm

Distance of image, q =?

Applying spherical mirror formula

e $$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

Putting the values, we get

$$\frac{1}{q} = \frac{1}{8} - \frac{1}{12}$$

$$= \frac{3-2}{24}$$

$$\frac{1}{q} = \frac{1}{24}$$

$$\boxed{q = 24\ cm}$$

Also, we know that

$$\frac{h_1}{h_0} = \frac{q}{p}$$

⇒ $$h_1 = \frac{q}{p} \times h_0$$

or $$h_1 = \frac{24}{12} \times 4$$

$$\boxed{h_1 = 8\ cm}$$

As, we are using convex lens and the object is placed away from the focal point, so the image will be.

[real, Inverted and magnified]

12.9 An object 10 cm high is placed at a distance of 20 cm from a concave lens of focal length 15 cm. Calculate the position and size of the image. Also state the nature of the image

Answer

Height of object, h_o = 10cm

Height of image, h_i =?

Distance of object, p =20cm

Distance of image, q =?

Applying the lens formula.

i.e $$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

Putting the values, we get

$$\frac{1}{q} = -\frac{1}{15} - \frac{1}{20}$$ (-ve f means formation of virtual image)

$$= \frac{-3-4}{60}$$

$$\frac{1}{q} = \frac{7}{60}$$

$$q = -60/7$$

$$\boxed{q = -8.57 \text{ cm}}$$

Also, we know that

$$\frac{h_1}{h_0} = \frac{q}{p}$$

$\Rightarrow$ $$h = \frac{q}{p} \times h_0$$

or $$h = \frac{8.57}{20} \times 10$$

$$\boxed{h = 4.28 \text{ cm}}$$

As, we are using convex lens and the object is placed away from the focal point, so the image will be.

[virtual, erect and diminished]

12.10 A convex lens of focal length 6 cm is to be used to form a virtual image three times the size of the object. Where must the lens be placed?

Answer

Focal length, f = 6 cm

As given: Height of image h_i = 3 (height of object)

$\Rightarrow$ $$h_1 = 3h_0$$

Distance of object, $p = ?$

As we know that

$$\frac{h_1}{h_0} = \frac{q}{p}$$

or $\frac{3h_1}{h_0} = \frac{q}{p}$

or $\frac{q}{p} = 3$

or $\boxed{q = 3p}$ (i)

Applying the lens formula.

i.e $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$\frac{1}{f} = \frac{1}{p} - \frac{1}{3p}$ (-ve sign is due to virtual image)

$\frac{1}{6} = \frac{3-1}{3p}$

$\frac{1}{6} = \frac{2}{3p}$

$3p = 12$

$\boxed{p = 4cm}$

12.11 A ray of light from air is incident on a liquid surface at an angle of incidence 35°. Calculate the angle of refraction if the refractive index of the liquid is 1.25. Also calculate the critical angle between the liquid air inter-face.

Answer

Incident angle, $\theta_i = 35°$

Refracted angle, $\theta_r = ?$

Refractive index, $n = 1.25$

Critical angle $\theta_c = ?$

As we know that

$n = \frac{\sin\theta_i}{\sin\theta_r}$

$\sin\theta_r = \sin^{-1}\left[\frac{\sin 35°}{1.25}\right]$

$\theta_r = 27.31°$

For critical angle,

$\sin\theta_c = \frac{1}{n}$ $\theta_c = \sin^{-1}\left[\frac{1}{n}\right]$

$\theta_c = \sin^{-1}\left[\frac{1}{1.25}\right]$

$\boxed{\theta_c = 53.13°}$

12.12 The power of a convex lens is 5D. At what distance the object should be placed from the lens so that its real and 2 times larger image is formed.

Answer

Power of lens, p = 5D

Distance of object, p =?

It is given that, $h_i = 2h_o$

(focal length f is also not given)

We know that,

$p = \frac{1}{f}$ (meters)

$f(\text{meters}) = \frac{1}{p}$

$= \frac{1}{5}$

$= 0.2$ m

or $\boxed{f = 20cm}$

Also, $\frac{h_i}{h_o} = \frac{q}{p}$

$\frac{2h_o}{h_o} = \frac{q}{p}$

$\frac{q}{p} = 2$

$q = 2p$

Applying the lens formula

e $\frac{1}{r} = \frac{1}{p} + \frac{1}{q}$

$\frac{1}{r} = \frac{1}{p} + \frac{1}{2p}$

$\frac{1}{20} = \frac{2+1}{2p}$

$\frac{1}{20} = \frac{3}{2p}$

$2p = 60$

$\boxed{p = 30cm}$

Numerical Problems

13.1 The charge of how many negatively charged particles would be equal to 100 µC. Assume charge on one negative particle is 1.6×10^{-19} C?

Answer

Number of negatively charged particles,

$n = ?$

Total charge, $q = 100 \mu C$

$= 100 \times 10^{-6} C$

Charge on one negative particle,

$e = 1.6 \times 10^{-19} C$

We know that,

$q = ne$

or $n = \frac{q}{e}$

Putting the values, we get

$n = \frac{100 \times 10^{-6}}{1.6 \times 10^{-19}}$

$n = 0.625 \times 10^{-5}$

or $\boxed{n = 6.25 \times 10^{13}}$

13.2 Two-point charges q_1 = 10 µC and q_2 = 5 µC are placed at distance of 150 cm. What will be the Coulomb's force between them? Also find the direction of the force.

Answer

Point charge, q_1 =10 μC

= 10×10^{-6} C

Point charge, q_2 = 5 μC

= 5×10^{-6}C

Distance between charges, r= 150 cm

= 1.5 m

Force, F =?

Electrostatic constant k = $9 \times 10^9 Nm^2C^{-2}$

According to Coulomb's law,

$$F = k\frac{q_1q_2}{r^2}$$

Putting the values, we get

$$F = \frac{9 \times 10^9 \times (10 \times 10^{-6}) \times (5 \times 10^{-6})}{(1.5)}$$

$F = 20 \times 10^{-2}$ N

or $\boxed{F = 0.2\ N}$

As both the point charges have same charge in nature the force will be repulsive

13 3 The force of repulsion between two identical positive charges is 0.8 N, when the charges are 0.1 m apart. Find the value of each charge.

Answer

Force, F = 0.8 N

Distance between charges 5T= 0.1 m

As the charges are identical. So, we can write,

$$q_1 = q_2 = q = ?$$

According to coulomb's law,

$$F = k\frac{q_1 q_2}{r^2}$$

or $$F = k\frac{q \times q}{r^2}$$

or $$F = k\frac{q^2}{r^2}$$

or $$q^2 = \frac{F \times r^2}{k}$$

Putting the values, we get

$$q^2 = \frac{0.8 \times (0.1)^2}{9 \times 10^9}$$

$$q^2 = 8.9 \times 10^{-13}$$

or $$q = 9.4 \times 10^{-7} C$$

So $$\boxed{q_1 = q_2 = 9.4 \times 10^{-7} C}$$

13.4 Two charges repel each other with a force of 0.1 N when they are 5 cm apart. Find the forces between the same charges when they are 2 cm apart.

Answer

Force, $F = 0.1$ N

Distance between charges, $r = 5$ cm

$= 0.05$ m

New force, $F' = ?$

New distance, $r' = 2$ cm

$= 0.02$ m

According to coulomb's law, when distance is 5 cm, we get,

$$F = k\frac{q_1 q_2}{r^2} \qquad (i)$$

Again, applying coulomb's law, when distance is 2 cm, we

$$F' = k\frac{q_1 q_2}{r'^2} \qquad (ii)$$

∴ (the charges are same in both the cases)

So equ (1) becomes

$k\,(q_1 q_2) = F \times r^2$ putting in equ. (2)

We get,

$$F' = \frac{F \times r^2}{r'^2}$$

Putting the values,

$$F' = \frac{0.1 \times (0.05)^2}{(0.02)^2}$$

$$\boxed{F' = 0.62\ N}$$

13.5 The potential at a point in an electric field is IO4 V. If a charge of + 100 μC is brought from infinity to this point. What would be the amount of work done on it?

Answer

Potential V = 10^4 volts

Charge, q = 100μC

= 100×10^{-6} C

Work done W =?

We know that,

$$V = \frac{W}{q}$$

or $W = V \times q$

Putting the values, we get

$W = IO^{4} \times 100 \times 10^{-6}$

$\boxed{\text{Work} = 1 \text{ Joule}}$

13.6 A point charge of + 2C is transferred from a point at potential 100 V to point at potential 50 V. What would be the energy supplied by the charge?

Answer

Potential at one point V_a = 100 volts

Potential at other point, V_b = 50 volts

Charge, q = 2C

Energy supplied E = 9

As we know,

Energy supplied = $q\ (V_a - V_b)$

Putting the values, we get

= 2 (100 - 50)

. $\boxed{\text{Energy supplied} = 100\text{J}}$

13.7 A capacitor holds 0.06 coulombs of charge when fully charged by a 9 volt battery. Calculate capacitance of the capacitor.

Answer

Charge, Q = 0.06C

Potential, V = 9V

Capacitance, C =?

We know that,

$$Q = CV$$

or $$C = \frac{Q}{V}$$

Putting the values, we get

$$C = \frac{0.06}{9}$$

or $$\boxed{C = 6.67 \times 10^{-3} F}$$

13.8 A capacitor holds 0.03 coulombs of charge when fully charged by a 6-volt battery. How much voltage would be required for it to hold 2 coulombs of charge?

Answer

Charge, $Q = 0.03\ C$

Potential, $V = 6\ V$

New charge stored, $Q' = 2C$

New potential, $V' = ?$

As we know that $Q = CV$ (i)

and $Q' = CV'$ (ii)

As capacitance is same in both the cases, So.

$$\frac{Q}{V} = \frac{Q}{V}$$

or $$V' = Q' \times \frac{V}{Q}$$

Putting the values, we get

$$V = \frac{2 \times 6}{0.03}$$

or $$\boxed{V = 400\ V}$$

13.9 Two capacitors of capacitances 6 µF and 12 µF are connected in series with 12 V battery. Find the equivalent capacitance of the combination. Find the charge and the potential difference across each capacitor.

Answer

Capacitance of 1st, capacitor, $C_1 = 6\ \mu F$

$= 6 \times 10^{-6}\ F$

Capacitance of 2nd capacitor, $C_2 = 12\ \mu F$

$= 12 \times 10^{-6}\ F$

Total voltage, $V = 12\ V$

Equivalent capacitance, $C_{eq} = ?$

Potential across 1st capacitor, $V_1 = ?$

Potential across 2nd capacitor, $V_2 = ?$

As we have the series combination, So

Charge across each capacitor, $Q_1 = Q_2 = Q = ?$

As we know,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} = \frac{1}{C_2}$$

Putting the values, we get

$$\frac{1}{C_{eq}} = \frac{1}{6\mu F} = \frac{1}{12\mu F}$$

$$\frac{1}{C_{eq}} = \frac{2\mu F + 1\mu F}{12\mu F}$$

$$\frac{1}{C_{eq}} = \frac{3}{12}\ \mu F$$

Also, we know that, $Q = C_{eq} V$

$= 4 \times 10^{-6} \times 12$

$= 48 \times 10^{-6} C$

Charge at each Capacitor, $Q_1 = Q_2 = 48\ \mu C$

For first Capacitor, $Q = C_1V$

$$V_1 = \frac{Q}{C_1}$$

or $$V_1 = \frac{48 \times 10^{-6}}{6 \times 10^{-6}}$$

$$\boxed{V_1 = 8\ V}$$

For second Capacitor, $Q = C_2V_2$

$$V_2 = \frac{Q}{C_2}$$

or $$V_1 = \frac{48 \times 10^{-6}}{12 \times 10^{-6}}$$

$$\boxed{V_2 = 4\ V}$$

13.10 Two capacitors of capacitances 6 μF and 12 μF are connected in parallel with a 12V battery. Find the equivalent capacitance of the combination. Find the charge and the potential difference across each capacitor.

Answer

Capacitance of 1st capacitor, $C_1 = 6\mu F$

$= 6 \times 10^{-6} F$

Capacitance of 2nd capacitor, $C_2 = 12\mu F$

$= 12 \times 10^{-6} F$

Total potential supplied, $V = 12\ V$

Equivalent capacitance, $C_{eq} = ?$

Charge across 1st, capacitor $Q_1 = ?$

Charge across 2nd capacitor, $Q_2 = ?$

As we have the parallel combination of capacitors So,

$$V_1 = V_2 = V = ?$$

As we know

$$C_{eq} = C + C_2$$

Putting the values, we get

$$C_{eq} = 6\mu F + 12\mu F$$

$$\boxed{C_{eq} = 18\mu F}$$

Potential across each capacitor will be the same as that of battery due to parallel

combination of capacitors.

$$\boxed{V_1 = V_2 = 12V}$$

Charge across 1st capacitor

$$Q = C_1 V$$

$$= 6 \times 10^{-6} \times 12$$

$$Q = 72 \times 10^{-6} C$$

$$\boxed{Q_1 = 72\mu C}$$

Charge across 2nd capacitor,

$$Q_2 = C_2 V$$

$$= 12 \times 10^{-6} \times 12$$

$$Q_2 = 144 \times 10^{-6} C$$

$$\boxed{Q_2 = 44\mu C}$$

We know that

$$1\ Kwh = 36 \times 105 J$$

Or $$1\ Wh = 3600\ J$$

Or $1J = \frac{1}{3600}$ Watt hours

$100J = \frac{1000}{3600}$ Watt hours

$\boxed{1000J = 0.28 \text{ Watt hour}}$

Numerical Problems

14.1 A current of 3mA is flowing through a wire for 1 minute. What is the charge flowing through the wire?

Answer

Current, I = 3 mA

= 3×10^{-3} A

Time, t = 1 min

= 60 sec

Charge, Q =?

As we know that,

I = Q/t, or Q = It

Putting the values, we get

$Q = 3 \times 10^{-3} \times 60$

$\boxed{Q = 180 \times 10^{-3}\ C}$

14.2 At 100,000 Ωm how much current flows through your body if you touch the terminals of a 12 V battery? If your skin is wet, so that your resistance is only 1000Ω, how much current would you receive from the same battery?

Answer

Resistance R = 10000 Ω

= 10^5 Ω

Current I =?

Voltage, v = 12 V

Resistance of wet skin $R' = 1000\ \Omega$

Current flowing through we skin, I' =?

i) For dry skin

applying ohm's law we have:

V=IR

Or, I = V/R

Putting the values, we get

$$I = \frac{12}{10^5}$$

$$\boxed{I = 1.2 \times 10^{-4}\ A}$$

ii) For wet skin

Again, applying ohm's law, we get,

$V' = I' R'$, or $I' = V/R'$

Putting the values, we get

I' =12/1000

or $\boxed{I' = 1.2 \times 10^{-2}\ A}$

14 3 The resistance of a conductor wire is 10 MΩ. If a potential difference of 100 V applied across its ends, then find the value of current passing through it in mA.

Answer

Resistance R = 10 M Ω

$= 10 \times 10^6\ \Omega$

Voltage, V = 100V

Current (mA) =I (mA)=?

Applying ohm's low we get:

$V = IR$

$I = V/R$

=> $I = 100/10^7$

$I = 10^{-5}$ A

As we know that, 1A = 10^3mA.

$\boxed{I = 0.01 \text{ mA}}$

14.4, By applying a potential difference of 10 V across a conductor a current of 1.5 A passes through it. How much energy would be obtained from the current in 2 minutes?

Answer

Voltage, V = 10V

Current, I = 1.5 A

Energy obtained =?

Time, t = 2 min

= 120 sec

As we know according to Joule's law, we get:

Energy = VIt

Putting the values, we get

Energy = 10 × 1.5 × 120

$\boxed{\text{Energy obtained} = 1800 \text{ J}}$

14.5 Two resistance of 2kΩ and 8kΩ are joined in series, if a 10 V battery is connected across the ends of this combination, find the following quantities:

a) The equivalent resistance of the parallel combination.

b) Current passing through each of the resistance.

c) Potential difference across each of the resistance.

Answer

First resistance, R = 2k Ω

= 2000 Ω

Second resistance, R2 = 8 KΩ

= 8000 Ω,

Voltage supplied, V = 12 V

Equivalent resistance, $R_{eq.}$ = ?

As the resistance are joined in series,

$I_1 = I_2 = I = ?$

Voltage across R_1, V_1 = ?

Voltage across R_2, V_2 = ?

We know that

$R_{eq} = R_1 + R_2$

=> = 2KΩ 8kΩ

Also, for current; applying ohm's law;

$V = IR_{eq}$

$I = V/R_{eq}$

= 12/10,000

I = 0.001 A

$I = 1mA$

$I = I_1 = I_2$, So $\boxed{I_2 = I_2 = 1\ mA}$

For voltage, again applying ohm's law,

1) For resistance R1,

=> $V_1 = R$,

$= 0.001 \times 2000$

$\boxed{V_1 = 2V}$

2) for resistance R2,

=> $V_2 = IR_2$

$= 0.001 \times 8000$

$\boxed{V_2 = 8V}$

14.6 Two resistance of 6Ω and 12 Ω are connected in parallel. A 6V battery is connected across its ends, find the values of the following quantities.

a) Equivalent resistance of the parallel combination.

b) Current passing through each of the resistance.

c) Potential difference across each of the resistance.

Answer

First resistance, $R_1 = 6k\ \Omega$

$= 6000\ \Omega$

Second resistance, $R_2 = 12k\ \Omega$

$= 12000\ \Omega$

Voltage supplied, $V = 6V$

Equivalent resistance, R_{eq} =?

Current through resistance, R_{eq} =?

Current through resistance, $R_1 I_1$ =?

Current through resistance, $R_2 I_2$ =?

As the combination is parallel, so, $V_1 = V_2 = V$ =?

We know that $1/R_{eq} = 1/R_1 + 1/R_2$

$$= \frac{1}{6k\,\Omega} + \frac{1}{12k\,\Omega}$$

$$= \frac{2k\,\Omega + 1k\,\Omega}{12k\,\Omega}$$

$$\frac{1}{R_{eq}} = \frac{3}{12}k\,\Omega$$

or $\boxed{R_{eq} = 4k\,\Omega}$

As For parallel combination, voltage remains the same, So $V = V_1 = V_2 = 12$ volts

Or $V_1 = V_2 = 12$ volts

For current applying ohm's law;

1) For resistance R_1,

$V = I_1R_1$

$I1 = V/R_1$

or $= 6/6000$

$I_1 = 10^{-3}A$

or $\boxed{I_1 = 1mA}$

2) For resistance R_2'

$V = I_2R_2$

$I_2 = V/R_2$

or $= 6/12000$

$I_2 = 5 \times 10^{-4}$ A

or $\boxed{I_2 = 0.5 \text{ mA}}$

14 7 An electric bulb is marked with 220V/ 100W. Find the resistance of the filament of the bulb. If the bulb is used 5 hours daily, find the energy in kilowatt-hour consumed by the bulb in one month (30 days).

Answer

Voltage, V = 220 V

Power, P = 100W

Resistance, R =?

Time, t = 5 × 30

= 150 hrs

Energy (kwh) =?

We know that

$P = V^2/R$

or $R = V^2/P$

Putting the values, we get

$R = (220)^2/110$

or $R = 484\ \Omega$

Also, we know that

Energy (KWh) = $\frac{\text{Watt} \times \text{time (hours)}}{1000}$

$= \frac{100 \times 150}{1000}$

$\boxed{\text{Energy consumed in KWh} = 15\ \text{KWh}}$

14.8 An incandescent light bulb with an operating resistance of 95 Ω is labelled "150 W". Is this bulb designed for use in a 120V circuit or a 220V circuit?

Answer

Resistance, $R = 95\ \Omega$

Power, $P = 150\ W$

Voltage, $V = ?$

We know that:

$P = V^2 / R$

or $V^2 = P \times R$

$V2 = 150 \times 95$

$V = 119.3\ V$

$\boxed{V = 120\ V}$

∴ (It has been designed for 120 V)

14.9 A house is installed with:

a)10 bulbs of 50 W each of which are used 5 hours daily.

b) 4 fans of 75 W each of which run 10 hours daily

c) One T.V of 250W which is used for 2 hours daily.

d) One electric iron of 1000 W which is used for 2 hours daily.

If the cost of one unit of electricity is Rs. 4. Find the monthly expenditure of electricity (one month = 30 days)

Answer.

a) Energy consumed by bulb (KWh) = $\frac{(10 \times 60) \times (5 \times 30)\ \text{hours}}{1000}$

$= \frac{600\ W \times 150\ hrs}{1000}$

= 90 KWh

b) Energy consumed by fans (KWh) = $\frac{(4 \times 75) \times (10 \times 30)\ \text{hours}}{1000}$

$= \frac{300\ W \times 300\ hrs}{1000}$

= 90 KWh

c) Energy consumed by T.V (KWh) = $\frac{(1 \times 250) \times (2 \times 30)\ \text{hours}}{1000}$

$= \frac{250\ W \times 60\ hrs}{1000}$

= 15 KWh

d) Energy consumed by electric iron (KWh) = $\frac{(1 \times 1000) \times (2 \times 30)\ \text{hours}}{1000}$

$= \frac{1000\ W \times 60\ hrs}{1000}$

= 60 KWh

Total energy consumed = (90 + 90 + 15 + 60)

= 255 KWh

Monthly expenditure of electricity = Total energy consumed x cost per unit

= 225 × 4

$\boxed{= \text{Rs. } 1020/-}$

14.10 A 100 W lamp bulb and a 4kW water heater are connected to a 250 V supply. Calculate (a) the current which flows in each appliance (b) the resistance of each appliance when is use.

Answer

Power of bulb, P_1 = 100 W

Power of water heater, P_2 = 4 KW

= 4000 W

Voltage, V = 250 V

Current through bulb, I_1 =?

Current through water heater, I2=?

Resistance of bulb, R_1=?

Resistance of water heater; R2=?

1) For bulb; we have, $P_1 = VI_1$,

$= I = P_1/V$

or I_1= 100/250

$\boxed{I = 0.4\text{ A}}$

For resistance, we apply ohm's law

= V= I R

or $R_1 = \frac{V}{I_1}$

$R = \frac{250}{0.4}$

$\boxed{R = 625\ \Omega}$

2) For water heater; we have;

$P_2 = VI_2$

Or $I_2 = P_2/V$

= 4000/250

$\boxed{I_2 = 16\text{ A}}$

And, $V = I_2R_2$

$R_2 = V/I_2$

$= 250/16$

$\boxed{R_2 = 15.6\ \Omega}$

14.11 A resistor of resistance 5.6Ω is connected across a battery of 3.0 V by means of wire of negligible resistance. A current of 0.5 A passes through the resistor. Calculate

a) power dissipated in the resistor

b) total power produced by the battery

c) Give the reason of difference between these two quantities

Answer

Resistance of resistor, $R = 5.6\ \Omega$

Voltage across battery, $V = 3$ V

Current through resistor, $I = 0.5$ A

Power dissipated in resistor, $P = ?$

Total power produced by battery, $P' = ?$

1) For Resistor R

We know that, $P = I^2 R$

$= (0.5)^2 \times 5.6$

$\boxed{P = 1.4\ W}$

2) For battery:

As the internal resistance of battery is not known so,

$V = IR'$

$R' = V/I$

Or $R' = 3/0.5$

$R' = 6\Omega$

It means that battery has resistance = R' - R = 6 - 5.6

$= 0.4\ \Omega$

So, power produced by the battery;

$P' = I^2 R$

$= (0.5)^2 \times 6$

$\boxed{P' = 1.5\ W}$

Numerical Problems

15.1 A transformer is needed to convert a mains 240 V supply into a 12 V supply. If there are 2000 turns on the primary coil, then find the number of turns on the secondary coil.

Answer

Input voltage, $V_p = 240$ V

Output voltage, $V_s = 12$ V

Turns on primary coil, $N_p = 2000$

Turns on secondary coil, $N_s = ?$

By the formula of turn ratio in transformer,

We have:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

or $$N_s = \frac{V_s}{V_p} \times N_p$$

Putting the values, we get

$$N_s = \frac{12}{240} \times 2000$$

$$\boxed{N_s = 100 \text{ turns}}$$

15.2 A step-up transformer has a turn ratio of 1 : 100. An alternating supply of 20 V is connected across the primary coil. What is the secondary voltage?

Answer

As the turn ratio is given, i.e. $N_p : N_s$ in step up transformer, or 1 : 100

It means that $\frac{N_s}{N_p} = \frac{100}{1}$

Also, input voltage, $V_p = 20$ V

Output voltage V_s =?

Applying the formula:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Putting the values, we get

$$\frac{V_s}{20} = \frac{100}{1}$$

$V_s = 20 \times 100$

$\boxed{V_s = 2000 \text{ volts}}$

15.3 A step-down transformer has a turns ratio of 1: 100. An A.C voltage of amplitude 170 V is applied to the primary. If the current in the primary is 1.0 mA, what is the current in the secondary?

Answer

As a step-down transformer is given; So.

$N_p : N_s$

It means that, 100 :1

Therefore, $\frac{N_s}{N_p} = \frac{1}{100}$

input A.C voltage, $V_p = 170$ V

Output A.C voltage, V_s =?

Input current, $I_p = 1$mA

$= 1 \times 10^{-3}$ A

Output current; I_s =?

We know that

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

or $$V_s = \frac{N_s}{N_p} \times V_p$$

putting the values, we get

$$V_s = \frac{1}{100} \times 170$$

or $$V_s = 1\,7\ \text{V}$$

Now, applying energy conservation formula

$$V_p I_p = V_s I_s$$

⇒ $$I_s = \frac{V_p I_p}{V_s}$$

Putting the values, we get

$$I_s = \frac{170 \times 10^{-3}}{1\,7}$$

or $$\boxed{I_s = 0\,1\text{A}}$$

15 1 A transformer, designed to convert the voltage from 240 V A.C. mains to 12 V, has 4000 turns on the primary coil. How many turns should be on the secondary coil? If the transformer were 100 % efficient, what current would flow through the primary coil when the current in the secondary coil was 0.4 A?

Answer

Input voltage, V_p = 240 V

Output voltage, V_s = 12 V

Input current I_p =? .

Output current, I_s = 0.4 A

Turns on primary coil, N_p = 4000

Turns on secondary coil, N_s =?

We know that

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

$\Rightarrow$ $$N_s = \frac{V_s}{V_p} \times N_p$$

Putting the values, we get

$$N_s = \frac{12}{240} \times 4000$$

or $$\boxed{N_s = 200 \text{ turns}}$$

Applying energy conservation formula

$$V_pI_p = V_sI_s$$

$\Rightarrow$ $$I_p = \frac{V_sI_s}{V_p}$$

Putting the values, we get

$$I_p = \frac{12 \times 0.4}{240}$$

or $$\boxed{I_p = 0.02A}$$

15.5 A power station generates 500 MW of electrical power which is fed to a transmission line. What current would flow in the transmission line if the input voltage is 250 KV?

Answer

Input power, P= 500 MW

$= 500 \times 10^6 W$

Input current, I_p =?

Input voltage, V_p =250 kv

$=250 \times 10^3 V$

We know that,

$$P = V_p I_p$$

$$\Rightarrow \quad I_p = \frac{P}{V_p}$$

Putting the values, we get

$$I_p = \frac{500 \times 10^6}{250 \times 10^3}$$

or $\quad I_p = 200A$

or $\quad \boxed{I_p = 2\times 10^3 A}$

15.6 The diagram shows a wind turbine which runs a 150-kW generator with an output voltage of 1000 V. the voltage is increased by transformer T_1 to 10,000 V for transmission to a town 5 km away through power lines with a total resistance of 2 Ω. Another transformer, T2, at the town reduces the voltage to 250 V. Assume that the transformers are 'Ideal'.

When the system is running at ful power:

a) What is the current in the power line?

b) What is the voltage drop and power loss along the power line? Also find the voltage at the input to the town transformer

Answer

Power given to transmission line, P= 150 kw

$$= 150 \times 10^3 \text{ W}$$

Output voltage from transformer T_1, V_s = 10000 V

Resistance in transmission line, R = 2 Ω

Current through transmission line I_s =?

a) We know that $P = V_s I_s$

or $I_s = \frac{P}{V_s}$

Putting the values, we get

$$I_s = \frac{150 \times 10^3}{10000}$$

$$\boxed{I_s = 15A}$$

b) i): Also, we know that

$$V = IR$$

$$\Rightarrow \quad V = 15 \times 2$$

$$\boxed{V = 30 \text{ volts}}$$

It shows that 30 V voltage will be drop along the power line

ii) Power along transmission line, P' =?

We know that,

$$P' = V$$

$$= 30 \times 15$$

$$\boxed{P' = 450 \text{ W}}$$

It shows that 450 W power will be lost along the transmission line.

iii) Voltage given to transmission line, = 10000 V

Voltage lost along transmission line = 30 V

So

Voltage at input of T_2 = 000-30

= 9970 V

$\boxed{V' = 9970\ V}$

Numerical Problems

18.1 The half-life of ^{16}N is 7.3s. A sample of this nuclide of nitrogen is observed for 29.2s. Calculate the fraction of the original radioactive isotope remaining after this time.

Answer

Half-life of $^{16}_{7}N$ = 7.3 s

Time observed = 29.2 s

After the time (half-life) 7.3 s = $\frac{1}{2}$ of sample will decay

After the time (2nd half-life),14.6 s = $\left(\frac{1}{2}\right)^2$

= $\frac{1}{4}$ th of the sample will be left

After the time (3rd half-life), 21.9 s = $\left(\frac{1}{2}\right)^3$

= $\frac{1}{8}$ th of the sample will be left

After the time (4th half-life) 29.2 s = $\left(\frac{1}{2}\right)^4$

= $\frac{1}{16}$ th of the sample will be left

[So, after 29 $\frac{1}{16}$ th fraction of the original radioactive isotope will remain]

18.2 Cobalt-60 is a radioactive element with half-life of 5.25 years. What fraction of the original sample will be left after 26 years?

Answer

Half-life of cobalt -60 = 5.25 years

Time observed = 26 years

1st half life After the time 5.25 years = $\frac{1}{2}$ of sample will decay.

2nd half-life: After the time 10.50 years $=\left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$th of the fraction will remain

3rd half-life: After the time 15.75 years $= \left(\frac{1}{2}\right)^3$

$= \frac{1}{8}$th of the fraction will remain

4th half-life: After the time 21 years$=\left(\frac{1}{2}\right)^4$

$= \frac{1}{16}$th of the fraction will remain

5th half-life: After the time 26 years $= \left(\frac{1}{2}\right)^5$

$= \frac{1}{32}$th of the fraction will remain

[So, after 26.25 years or approximately 26 years, 1/32th fraction of the original sample will be left]

18.3 Carbon-14 has a half-life of 5730 years. How long will it take for the quantity of carbon-14 in a sample to drop to one-eighth of the initial quantity?

Answer

Half-life of carbon-14 = 5730 years

Time required for 1st half-life = 5730 years

Time required for 2nd half-life = 1.14×10^4 years

Time required for 3^{rd} half-life = 1.72×10^4 years

[It means that after 3^{rd} half-life 1 / 8th of the initial quantity is left

and 1.72×10^4 years' time is required for this decay.]

18.4 Technetium -99m is a radioactive element and is used to diagnose brain, thyroid, liver and kidney diseases. This element has half-life of 36 minutes. If there is 200 mg of this technetium present, how much will be left in six hours.

Answer

Half-life of Technetium - 99 = 36 minutes

Mass of Technetium 99 = 200 mg

After 36 min = 100mg of Technetium will be left

Within 30 = 50 mg of Technetium will be left

Within 24 = 25 mg of Technetium will be left

Within 18 = 12.5 mg of Technetium will be left

Within 12 = 6.25 mg of Technetium will be left

Within 6 = 3.12 mg of Technetium will be left

[it means that within 6 minutes 3.12 mg of Technetium may decay.]

18.5 Half-life of a radioactive element is 10 minutes. If the initial count rate is 368 counts per minute, find the time for which count rates reaches 23 counts per minute.

Answer

Half life of radioactive element = 10min

Initial count rate = 368 c/min

After 10 min = 184 c/min

After 20 min = 92 c/min

After 30 min = 46 c/min

After 40 min = 23 c/min

[So, after 40 minutes we have 23 counts per minute as decay rate]

18.6 In an experiment to measure the half-life of a radioactive element, the following results were obtained:

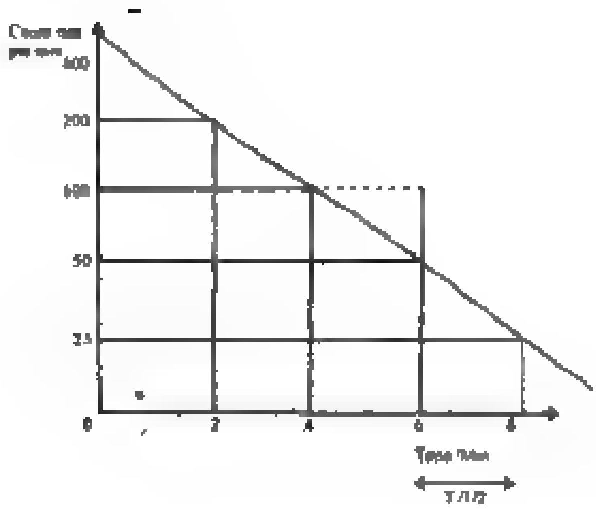
Count rate	400	200	100	50	25
Time (in minutes)	0	2	4	6	8

Plot a graph between the count rate and time in minutes. Measure the value of the half life of the element from the graph.

Answer

It clear from the graph that half live of the radioactive element is 2 minutes.

After each two-minute $\frac{1}{2}$ of Ae remaining specimen is decayed



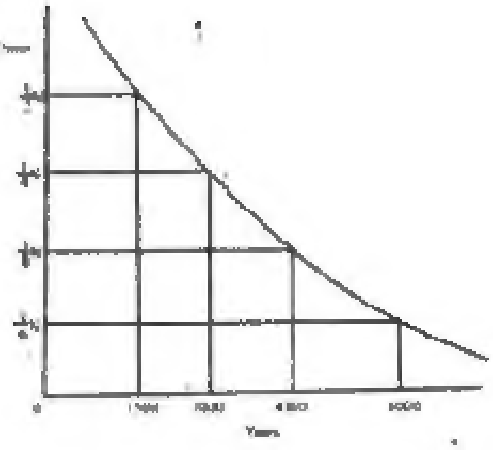
18.7 A sample of certain radioactive element has at half-life of 1500 years. If it has an activity of 32000 counts per hour at the present time then plot a graph of the activity of this sample over the period in which it will reduce to 1/16 of its present value.

Answer

Half 1 life of radioactive element= 1500 years

Rate of activity = 32000 c/min-

It is clear from the graph that 6000 years are required to decay the radioactive element in such a way that only $1/16^{th}$ of this specimen is left



18.8 Half-life of radioactive element was found to be 4000 years. The count rates per minute for 8 successive hours were (·>und to be 270, 280, 300, 310, 285, 290, 305, 312. What does the variation in count rates show? Plot graph between the count rates and time in hours. Why the graph is a straight line rather than an exponential?

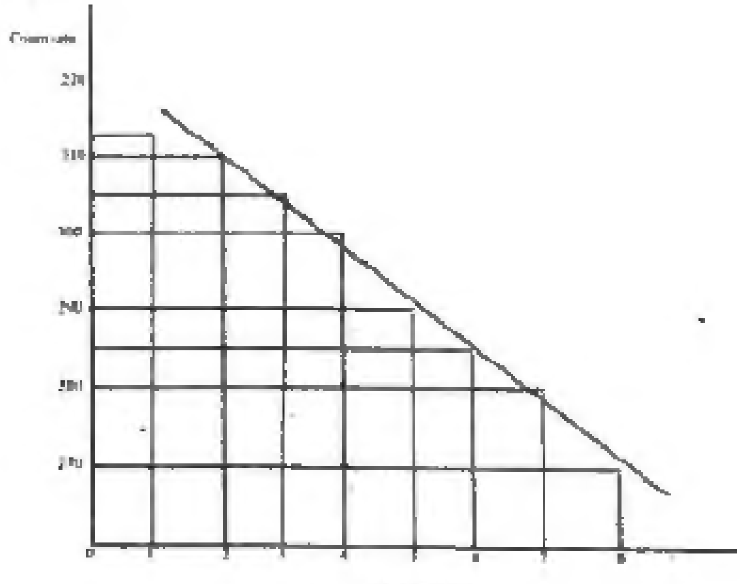
Answer

Half-life of radioactive element = 4000 years

Time for activity = 8 hours.

Variation in count rate shows the random nature of radioactive decay.

Graph as shown below.



[Graph is almost horizontal line rather than exponential curve which is due to long half-life as compared to 8]

18.9 Ashes from a campfire deep in a cave show carbon-14 activity of only one-eighth the activity of fresh wood. How long ago was that campfire made?

Answer

Activity of carbon - 14 = 1/8 of activity of wood

As we know that:

The half-life of carbon - 14 = 5730 years

Time for carbon - 14 to decay ½ of activity

Time for carbon - 14 to decay $1/4^{th}$ of activity

of fresh wood = 11460 years

Time for carbon - 14 to decay $\frac{1}{8}^{th}$ of activity

of fresh wood = 17190 years.

17190 years